Plane wave scattering induced resonant modes of cubic resonator

H. Yoo and A. Gopinath

Plane wave scattering from a cubic resonator is calculated by solving the combined field integral equation with the Rao-Wilton-Glisson basis functions and the moment method. High dielectric cube calculations show that magnetic and electric dipoles are found at the first and the second modes, respectively, whereas the third resonant mode has simultaneously strong electric and magnetic fields at the same resonant frequency. Furthermore, an analysis of a magnetodielectric cube is discussed.

Introduction: Dielectric resonators have been investigated in various scientific and engineering research areas including antenna designs and microwave integrated circuits because of their advantages such as high radiation efficiency, ease of implementation and numerous possible applications [1–3]. To analyse dielectric resonators, the well-known method of moments and the combined field integral equation (CFIE) [4] with the Rao-Wilton-Glisson basis functions [5] are used to calculate the scattered fields. For the high dielectric cube, a theoretical analysis shows that the magnetic and electric dipoles are found at the first and the second modes, respectively, whereas the third resonant mode has simultaneously strong electric and magnetic fields at the same resonant frequency. In addition, a magnetodielectric resonator with the same dimensions as the high dielectric cube is also studied. An analysis of the magnetodielectric cube results in almost identical internal field distributions at the first two resonant modes with different frequencies as the high dielectric cube, but no simultaneous strong magnetic and electric resonances are found at the third resonant frequency. In our previous study [6], the metamaterial structure composed of high dielectric resonators in a low dielectric substrate is analysed to improve antenna gain, but there is no specific analysis of resonant modes. In this Letter, plane wave scattering resonant modes of cubic resonators are studied including high dielectric and magnetodielectric resonators. To the best of our knowledge, there are no previous analyses of high dielectric resonator structures at the third resonant mode. This result of this analysis suggests the possibility of developing a resonator array that could be used as a negative-index material (or metamaterial) [7–9]. Well established and comprehensively investigated spheres also can be analysed, but they are not appropriate in terms of a manufacturing process.

Method: Before plane wave scattering from the cubic resonators can be calculated using the CFIE method [4], a resonant frequency should be specified. Analytically, a rectangular resonant cavity with conducting walls has its resonant frequencies given by [10]

\[
\frac{1}{2\pi}\sqrt{\frac{\varepsilon_r n^2}{\mu_r m^2}} + \frac{\eta^2}{W^2} + \frac{\eta^2}{L^2} + \frac{\eta^2}{H^2}
\]

(1)

where \( m = 0, 1, 2, \ldots; n = 0, 1, 2, \ldots; p = 1, 2, 3, \ldots \) and \( m = n = 0 \) for the TE mode, \( m = 1, 2, 3, \ldots; n = 1, 2, 3, \ldots \) and \( p = 0, 1, 2, \ldots \) for the TM mode, respectively. \( W, H \) and \( L \) represent the length in the \( yz \), \( xz \) and \( xy \) dimensions of the rectangular cavity, respectively. For a cubic cavity where \( W = H = L \), the lowest order modes TE011, TE101 and TM110 are degenerate, as they have the same resonant frequency and field distribution. The high dielectric cubic resonator may be assumed to have open circuit boundaries, in which case these degenerate modes change and the TE modes become TM and the TM mode becomes TE. Thus, (1) with \( W = H = L \) may also be used for the resonant frequencies of the high dielectric cubic resonators.

Plane wave scattering from high dielectric and magnetodielectric objects is obtained from the solution of the CFIE to determine the internal magnetic and electric field distribution and far-field radiation patterns. As shown in Fig. 1, the scatterer is located in free space and is excited by a plane wave propagating along the \( x \)-direction with its polarised electric field along the \( y \)-axis. Note that three different planes, the \( yz \), \( xz \) and \( xy \)-planes, are used in internal field calculations. Since the scattered electric and magnetic fields are complex numbers, their scattered normalised magnitude is used for internal field distributions.

Results: A high dielectric cube with relative permittivity \( \varepsilon_r = 30 \) is also studied as a resonator. The dimensions are \( W = H = L = 8 \) mm, which is less than \( 0.73 a_0 \), where \( a_0 \) is the wavelength in free space at the first resonant frequency. According to (1), with a rectangular cavity, the first two resonant frequencies are approximately 4.84 and 5.9 GHz. In the field calculations, internal field distributions from 5 to 6 GHz provide almost the same field patterns with the exception of field strength. As shown in Fig. 2, the magnetic fields are at the maximum value of 0.15 at 5.5 GHz, whereas the maximum electric field is 6 at 7.5 GHz. Magnetic and electric dipoles because of strong displacement currents are obtained at the first and second resonant modes, and the radiation patterns in the far field plotted in Figs 3a and b confirm these characteristics of the dipole. At 8.21 GHz, which is slightly less than the resonant frequency of 8.38 GHz from (1), the cubic high dielectric resonator has simultaneously strong electric and magnetic resonances, as shown in Fig. 4. The maximum magnetic field strength is approximately 0.4, which is almost three times that at the first resonance. Moreover, the maximum electric field strength is five times higher than maximum electric field strength at the second resonance. These simultaneously strong electric and magnetic fields show characteristics of a quadrupole, as shown in Fig. 3c. These properties produce behaviour similar to that of a two-dipole array. At the same high dielectric cube dimensions \( (W = H = L = 8) \) mm, the relative permittivity \( \varepsilon_r = 16 \) and permeability \( \mu_r = 10 \) were used to analyse a magnetodielectric cube. The value of \( \sqrt{\mu_r \varepsilon_r} \) is much higher than that of the dielectric cube, and the first three resonant frequencies from (1), which are relatively close to each other, are approximately 2.1, 2.57 and 3.31 GHz. In the scattering calculations, magnetic field distributions with low strength are obtained as shown in Figs. 5a–c at 2 GHz. These distributions are maintained up to nearly 3.18 GHz, and have maximum field strength of 0.14 at 3.125 GHz. The electric dipole slightly above this frequency is shown in Figs. 5d–f at 3.25 GHz. However, simultaneous magnetic and electric dipoles cannot be obtained through the scattered field calculations. Since the dipole shape shown in Fig. 5 is slightly tilted, the far-field radiation patterns are also shifted, as shown in Fig. 6.
Conclusions: Plane wave scattering from a cubic high dielectric resonator has been discussed to find resonant modes. The scattered magnetic and electric fields in the high dielectric resonator and the far-field radiation patterns show that magnetic and electric dipoles are found at the first and second modes, respectively, whereas strong electric and magnetic fields are simultaneously obtained at the third resonant mode. In addition, a magnetodielectric resonator with identical dimensions is also studied and compared with the high dielectric resonators. The magnetodielectric cube has similar behaviours at the first and second resonances; however, no simultaneous strong magnetic and electric resonances are obtained at the third resonant frequency.

Acknowledgment: This work was supported by the 2013 Research Fund of the University of Ulsan.

© The Institution of Engineering and Technology 2013
22 April 2013
doi: 10.1049/el.2013.1195
One or more of the Figures in this Letter are available in colour online.

H. Yoo (Department of Biomedical Engineering, School of Electrical Engineering, University of Ulsan, Ulsan, Republic of Korea)
E-mail: hsyoo@ulsan.ac.kr
A. Gopinath (Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN, USA)

References